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Determining SUSY parameters in chargino pair-production in e^+e^- collisions

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Abstract. In most supersymmetric theories, charginos $\tilde{\chi}^\pm_{1,2}$, mixtures of charged color-neutral gauginos and higgsinos, belong to the class of the lightest supersymmetric particles. They are easy to observe at e^+e^- colliders. By measuring the total cross sections and the left–right asymmetries with polarized electron beams in $e^+e^- \to \tilde{\chi}^-_i \tilde{\chi}^+_j$ $[i, j = 1, 2]$, the chargino masses and the gaugino–higgsino mixing angles can be determined. From these observables the fundamental SUSY parameters can be derived: the SU(2) gaugino mass M_2, the modulus $|\mu|$ and $\cos\Phi_\mu$ of the higgsino mass parameter, and $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two neutral Higgs doublet fields. The solutions are unique; the CP-violating phase Φ_μ can be determined uniquely by analyzing effects due to the normal polarization of the charginos.

1 Introduction

In supersymmetric theories, the spin-1/2 partners of the W bosons and the charged Higgs bosons, $\tilde{W}^\pm$ and $\tilde{H}^\pm$, mix to form chargino mass eigenstates $\tilde{\chi}^\pm_{1,2}$. The chargino mass matrix [1] is given in the $(\tilde{W}^-, \tilde{H}^-)$ basis by

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \cos\beta \\ \sqrt{2}m_W \sin\beta & \mu \end{pmatrix} \tag{1}$$

which is built up by the fundamental supersymmetry (SUSY) parameters: the SU(2) gaugino mass M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. In CP-noninvariant theories, the gaugino mass M_2 and the higgsino mass parameter μ can be complex. However, by reparametrization of the fields, M_2 can be assumed real and positive without loss of generality so that the only non-trivial invariant phase is attributed to μ:

$$\mu = |\mu| \mathrm{e}^{i\Phi_\mu} \tag{2}$$

The angle Φ_μ can vary between 0 and 2π. Once charginos are discovered, the experimental analysis of their properties, production and decay mechanisms will therefore reveal the basic structure of the underlying supersymmetric theory.

Charginos are produced in e^+e^- collisions, either in diagonal or in mixed pairs [2]–[6]. In the present analysis, we will focus on all combinations of chargino pairs $\tilde{\chi}^\pm_{1,2}$ in e^+e^- collisions:

$$e^+e^- \;\to\; \tilde{\chi}^-_i \;\tilde{\chi}^+_j \quad [i, j = 1, 2]$$

If the collider energy is sufficient to produce the two chargino states in pairs, the underlying fundamental SUSY parameters, $M_2, |\mu|$ and $\tan\beta$, can be extracted *unambiguously* from the masses $m_{\tilde{\chi}^\pm_{1,2}}$, the total production cross sections, and the left-right (LR) asymmetries with polarized electron beams, while the phase Φ_μ is determined up to a twofold ambiguity $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$. [This ambiguity can be resolved by measuring manifestly CP-noninvariant observables, see [7], related to the normal polarization of the charginos.]

This analysis of the chargino sector is independent of the structure of the neutralino sector, which is potentially more complex than the form encountered in the Minimal Supersymmetric Standard Model (MSSM). The structure of the chargino sector, by contrast, is isomorphic to the form of the MSSM for a large class of supersymmetric theories.

Moreover, from the energy distribution of the final particles in the decay of the lightest chargino, the mass of the lightest neutralino can be measured; this allows us to determine the other U(1) gaugino mass parameter M_1 if this parameter is real. If not, additional information on the phase of M_1 must be derived from observables involving the heavier neutralinos.

In summary: if the chargino/neutralino sector is CP-invariant, all fundamental gaugino parameters can be derived from the masses and cross sections of the chargino sector, supplemented by the mass of the lightest neutralino. In CP-noninvariant theories, the phase of μ can be determined up to a twofold ambiguity by measuring CP-even observables; the ambiguity can be resolved by analyzing manifestly CP-noninvariant observables. The phase of M_1 can only be obtained by exploiting observables involving heavier neutralino states.

The analysis will be based strictly on low-energy SUSY. Once these basic parameters have been extracted experimentally, they may be confronted, for instance, with the ensemble of relations predicted in Grand Unified Theories. The paper will be divided into six parts. In Sect. 2 we recapitulate the central elements of the mixing formalism for the charged gauginos and higgsinos. In Sect. 3 the cross sections for chargino production, the left–right asymmetries, and the polarization vectors of the charginos are given. In Sect. 4 we describe a phenomenological analysis based on a specific mSUGRA scenario to exemplify the procedure for extracting the fundamental SUSY parameters in a model-independent way. In Sect. 5 we briefly comment on the possibility of extracting the U(1) gaugino mass M_1 from the lightest neutralino mass measured in the decay $\tilde{\chi}_1^\pm \to W^\pm + \tilde{\chi}_1^0$. Conclusions are given in Sect. 6.

2 Mixing formalism

Since the chargino mass matrix $\mathcal{M}_C$ is not symmetric, two different unitary matrices acting on the left- and right-chiral $(\tilde{W},\tilde{H})$ states are needed to diagonalize the matrix:

$$U_{L,R}\begin{pmatrix}\tilde{W}^-\\ \tilde{H}^-\end{pmatrix}_{L,R}=\begin{pmatrix}\tilde{\chi}_1^-\\ \tilde{\chi}_2^-\end{pmatrix}_{L,R} \tag{3}$$

The unitary matrices U_L and U_R can be parametrized in the following way [7]:

$$U_L=\begin{pmatrix}\cos\phi_L & \mathrm{e}^{-i\beta_L}\sin\phi_L\\ -\mathrm{e}^{i\beta_L}\sin\phi_L & \cos\phi_L\end{pmatrix}$$
$$U_R=\begin{pmatrix}\mathrm{e}^{i\gamma_1} & 0\\ 0 & \mathrm{e}^{i\gamma_2}\end{pmatrix}\begin{pmatrix}\cos\phi_R & \mathrm{e}^{-i\beta_R}\sin\phi_R\\ -\mathrm{e}^{i\beta_R}\sin\phi_R & \cos\phi_R\end{pmatrix} \tag{4}$$

The eigenvalues $m^2_{\tilde{\chi}^\pm_{1,2}}$ are given by

$$m^2_{\tilde{\chi}^\pm_{1,2}}=\frac{1}{2}\left[M_2^2+|\mu|^2+2m_W^2\mp\Delta_C\right] \tag{5}$$

with Δ_C involving the phase Φ_μ:

$$\Delta_C=\sqrt{(M_2^2-|\mu|^2)^2+4m_W^4\cos^2 2\beta+4m_W^2(M_2^2+|\mu|^2)+8m_W^2M_2|\mu|\sin 2\beta\cos\Phi_\mu} \tag{6}$$

The quantity Δ_C determines the difference of the two chargino masses: $\Delta_C=m^2_{\tilde{\chi}_2^\pm}-m^2_{\tilde{\chi}_1^\pm}$. The four phase angles $\{\beta_L,\beta_R,\gamma_1,\gamma_2\}$ are not independent but can be expressed in terms of the invariant angle Φ_μ:

$$\begin{aligned}
\tan\beta_L&=-\frac{\sin\Phi_\mu}{\cos\Phi_\mu+\frac{M_2}{|\mu|}\cot\beta}\\
\tan\beta_R&=+\frac{\sin\Phi_\mu}{\cos\Phi_\mu+\frac{M_2}{|\mu|}\tan\beta}\\
\tan\gamma_1&=+\frac{\sin\Phi_\mu}{\cos\Phi_\mu+\frac{M_2(m^2_{\tilde{\chi}_1^\pm}-|\mu|^2)}{|\mu|m_W^2\sin 2\beta}}\\
\tan\gamma_2&=-\frac{\sin\Phi_\mu}{\cos\Phi_\mu+\frac{M_2m_W^2\sin 2\beta}{|\mu|(m^2_{\tilde{\chi}_2^\pm}-M_2^2)}}
\end{aligned} \tag{7}$$

All four phase angles vanish in CP-invariant theories for which $\Phi_\mu\to 0$ or π. The rotation angles ϕ_L and ϕ_R satisfy the relations:

$$\begin{aligned}
\cos 2\phi_L&=-\frac{M_2^2-|\mu|^2-2m_W^2\cos 2\beta}{\Delta_C}\\
\sin 2\phi_L&=-\frac{2m_W\sqrt{M_2^2+|\mu|^2+(M_2^2-|\mu|^2)\cos 2\beta+2M_2|\mu|\sin 2\beta\cos\Phi_\mu}}{\Delta_C}
\end{aligned}$$

and

$$\begin{aligned}
\cos 2\phi_R&=-\frac{M_2^2-|\mu|^2+2m_W^2\cos 2\beta}{\Delta_C}\\
\sin 2\phi_R&=-\frac{2m_W\sqrt{M_2^2+|\mu|^2-(M_2^2-|\mu|^2)\cos 2\beta+2M_2|\mu|\sin 2\beta\cos\Phi_\mu}}{\Delta_C}
\end{aligned} \tag{8}$$

As a consequence of possible field redefinitions, the parameters $\tan\beta$ and M_2 can be chosen real and positive.

The fundamental SUSY parameters M_2, $|\mu|$, $\tan\beta$ and the phase parameter $\cos\Phi_\mu$ can be extracted from the chargino $\tilde{\chi}^\pm_{1,2}$ parameters: the masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the two mixing angles ϕ_L and ϕ_R of the left- and right-chiral components of the wave function. These mixing angles are physical observables and they can be measured, as well as the chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$, in the processes $e^+e^-\to\tilde{\chi}_i^-\tilde{\chi}_j^+$ $[i,j=1,2]$.

The two angles ϕ_L and ϕ_R and the nontrivial phase angles $\{\beta_L,\beta_R,\gamma_1,\gamma_2\}$ define the couplings of the chargino-chargino-Z vertices and the electron-sneutrino-chargino vertex:

$$\begin{aligned}
\langle\tilde{\chi}^-_{1L}|Z|\tilde{\chi}^-_{1L}\rangle&=-\frac{e}{s_Wc_W}\left[s_W^2-\frac{3}{4}-\frac{1}{4}\cos 2\phi_L\right]\\
\langle\tilde{\chi}^-_{1L}|Z|\tilde{\chi}^-_{2L}\rangle&=+\frac{e}{4s_Wc_W}\mathrm{e}^{-i\beta_L}\sin 2\phi_L\\
\langle\tilde{\chi}^-_{2L}|Z|\tilde{\chi}^-_{2L}\rangle&=-\frac{e}{s_Wc_W}\left[s_W^2-\frac{3}{4}+\frac{1}{4}\cos 2\phi_L\right]\\
\langle\tilde{\chi}^-_{1R}|Z|\tilde{\chi}^-_{1R}\rangle&=-\frac{e}{s_Wc_W}\left[s_W^2-\frac{3}{4}-\frac{1}{4}\cos 2\phi_R\right]\\
\langle\tilde{\chi}^-_{1R}|Z|\tilde{\chi}^-_{2R}\rangle&=+\frac{e}{4s_Wc_W}\mathrm{e}^{-i(\beta_R-\gamma_1+\gamma_2)}\sin 2\phi_R\\
\langle\tilde{\chi}^-_{2R}|Z|\tilde{\chi}^-_{2R}\rangle&=-\frac{e}{s_Wc_W}\left[s_W^2-\frac{3}{4}+\frac{1}{4}\cos 2\phi_R\right]
\end{aligned}$$

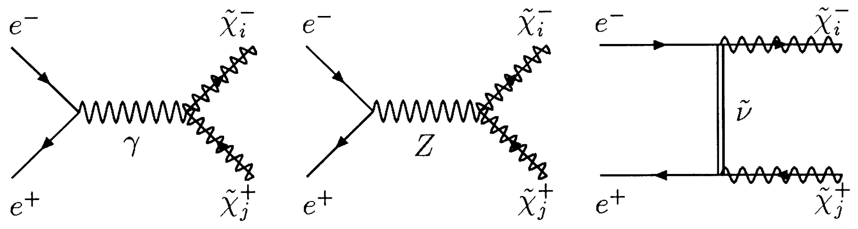

Fig. 1. The three exchange mechanisms contributing to the production of chargino pairs $\tilde{\chi}_i^- \tilde{\chi}_j^+$ in e^+e^- annihilation

$$\langle \tilde{\chi}_{1R}^- | \tilde{\nu} | e_L^- \rangle = -\frac{e}{s_W} \mathrm{e}^{i\gamma_1} \cos\phi_R$$
$$\langle \tilde{\chi}_{2R}^- | \tilde{\nu} | e_L^- \rangle = +\frac{e}{s_W} \mathrm{e}^{i(\beta_R+\gamma_2)} \sin\phi_R \tag{9}$$

where $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$. The coupling to the higgsino component, being proportional to the electron mass, has been neglected in the sneutrino vertex; the sneutrino couples only to left-handed electrons. Note that the CP-noninvariant phase Φ_μ enters the vertices through the phase angles which have been expressed in terms of the fundamental SUSY parameters in (7). Since the photon-chargino vertex is diagonal, it does not depend on the mixing angles:

$$\langle \tilde{\chi}_{iL,R}^- | \gamma | \tilde{\chi}_{jL,R}^- \rangle = e\delta_{ij} \tag{10}$$

The parameter e is the electromagnetic coupling which will be taken at an effective scale identified with the c.m. energy $\sqrt{s}$.

3 Chargino pair-production

The process $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+$ is generated by the three mechanisms shown in Fig. 1: s-channel γ and Z exchanges, and t-channel $\tilde{\nu}$ exchange. The transition matrix element, after a Fierz transformation of the $\tilde{\nu}$-exchange amplitude,

$$T\left(e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+\right) = \frac{e^2}{s} Q_{\alpha\beta} \left[\bar{v}(e^+)\gamma_\mu P_\alpha u(e^-)\right] \times \left[\bar{u}(\tilde{\chi}_i^-)\gamma^\mu P_\beta v(\tilde{\chi}_j^+)\right] \tag{11}$$

can be expressed in terms of four bilinear charges, defined by the chiralities $\alpha, \beta = L, R$ of the associated lepton and chargino currents

(i) $\underline{\tilde{\chi}_1^- \tilde{\chi}_1^+}$

$$\begin{aligned}
Q_{LL} &= 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L \right) \\
Q_{LR} &= 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R \right) \\
&\quad + \frac{D_{\tilde{\nu}}}{4s_W^2}(1 + \cos 2\phi_R) \\
Q_{RL} &= 1 + \frac{D_Z}{c_W^2} \left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L \right) \\
Q_{RR} &= 1 + \frac{D_Z}{c_W^2} \left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R \right)
\end{aligned} \tag{12}$$

(ii) $\underline{\tilde{\chi}_1^- \tilde{\chi}_2^+}$

$$\begin{aligned}
Q_{LL} &= \frac{D_Z}{4s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \mathrm{e}^{-i\beta_L} \sin 2\phi_L \\
Q_{LR} &= \frac{D_Z}{4s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \mathrm{e}^{-i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R \\
&\quad + \frac{D_{\tilde{\nu}}}{4s_W^2} \mathrm{e}^{-i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R \\
Q_{RL} &= \frac{D_Z}{4c_W^2} \mathrm{e}^{-i\beta_L} \sin 2\phi_L \\
Q_{RR} &= \frac{D_Z}{4c_W^2} \mathrm{e}^{-i(\beta_R - \gamma_1 + \gamma_2)} \sin 2\phi_R
\end{aligned} \tag{13}$$

(iii) $\underline{\tilde{\chi}_2^- \tilde{\chi}_2^+}$

$$\begin{aligned}
Q_{LL} &= 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \left(s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_L \right) \\
Q_{LR} &= 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2}) \left(s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_R \right) \\
&\quad + \frac{D_{\tilde{\nu}}}{4s_W^2}(1 - \cos 2\phi_R) \\
Q_{RL} &= 1 + \frac{D_Z}{c_W^2} \left(s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_L \right) \\
Q_{RR} &= 1 + \frac{D_Z}{c_W^2} \left(s_W^2 - \frac{3}{4} + \frac{1}{4}\cos 2\phi_R \right)
\end{aligned} \tag{14}$$

The first index in $Q_{\alpha\beta}$ refers to the chirality of the $e^\pm$ current, the second index to the chirality of the $\tilde{\chi}_i^-/\tilde{\chi}_j^+$ current. The $\tilde{\nu}$ exchange only affects the LR chirality charge while all other amplitudes are built up by γ and Z exchanges only. $D_{\tilde{\nu}}$ denotes the sneutrino propagator $D_{\tilde{\nu}} = s/(t - m_{\tilde{\nu}}^2)$, and D_Z the Z propagator $D_Z = s/(s - m_Z^2 + im_Z\Gamma_Z)$; the non-zero Z width can in general be neglected for the energies considered in the present analysis so that the charges are rendered complex in the present Born approximation only through the CP-noninvariant phases.

For the sake of convenience we also introduce the eight quartic charges [8] defined in Table 1. These charges correspond to the eight independent helicity amplitudes describing the chargino production processes for massless electrons/positrons.

The charges Q_1 to Q_4 are manifestly parity-even, i.e., invariant under space reflection; Q_1' to Q_4' are parity-odd.

Table 1. Quartic charges determining the cross section and polarization vectors in pair-production of charginos in e^+e^- collisions. Detailed comments are given in the text

$\mathcal{P}$	$\mathcal{CP}$	Quartic Charges
		$Q_1 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LL}\rvert^2 + \lvert Q_{RL}\rvert^2 + \lvert Q_{LR}\rvert^2\right]$
even	even	$Q_2 = \frac{1}{2}\mathrm{Re}\,[Q_{RR}Q^*_{RL} + Q_{LL}Q^*_{LR}]$
		$Q_3 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LL}\rvert^2 - \lvert Q_{RL}\rvert^2 - \lvert Q_{LR}\rvert^2\right]$
	odd	$Q_4 = \frac{1}{2}\mathrm{Im}\,[Q_{RR}Q^*_{RL} + Q_{LL}Q^*_{LR}]$
		$Q'_1 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{RL}\rvert^2 - \lvert Q_{LR}\rvert^2 - \lvert Q_{LL}\rvert^2\right]$
odd	even	$Q'_2 = \frac{1}{2}\mathrm{Re}\,[Q_{RR}Q^*_{RL} - Q_{LL}Q^*_{LR}]$
		$Q'_3 = \frac{1}{4}\left[\lvert Q_{RR}\rvert^2 + \lvert Q_{LR}\rvert^2 - \lvert Q_{RL}\rvert^2 - \lvert Q_{LL}\rvert^2\right]$
	odd	$Q'_4 = \frac{1}{2}\mathrm{Im}\,[Q_{RR}Q^*_{RL} - Q_{LL}Q^*_{LR}]$

The charges Q_1 to Q_3 and Q'_1 to Q'_3 are CP invariant[1] while Q_4 and Q'_4 change sign under CP transformations. The CP invariance of Q_2 and Q'_2 can easily be shown by noting that

$$\begin{aligned}&\cos(\beta_L - \beta_R + \gamma_1 - \gamma_2)\sin 2\phi_L \sin 2\phi_R \\ &= \frac{m^2_{\tilde{\chi}^\pm_1} + m^2_{\tilde{\chi}^\pm_2}}{2m_{\tilde{\chi}^\pm_1} m_{\tilde{\chi}^\pm_2}}(1-\cos 2\phi_L \cos 2\phi_R) - \frac{2m^2_W}{m_{\tilde{\chi}^\pm_1} m_{\tilde{\chi}^\pm_2}}\end{aligned} \tag{15}$$

Thus, all the cross sections $e^+e^- \to \tilde{\chi}^-_i \tilde{\chi}^+_j$ for any combination of pairs (ij) depend only on $\cos 2\phi_L$ and $\cos 2\phi_R$. For polarized electron beams the sums and differences of the quartic charges are restricted to either L or R components (first index) of the $e^\pm$ currents.

The measurement of the quartic charges Q_1 to Q'_3 in the total cross sections and left–right asymmetries for equal and mixed chargino pair-production allows us to extract the two terms $\cos 2\phi_L$ and $\cos 2\phi_R$ unambiguously as will be demonstrated explicitly in the following section.

The CP-noninvariant charges Q_4 and Q'_4 vanish for equal chargino pairs $\tilde{\chi}^-_1\tilde{\chi}^+_1$ and $\tilde{\chi}^-_2\tilde{\chi}^+_2$. They can be determined only by measuring observables related to the normal components of the $\tilde{\chi}^\pm_{1,2}$ polarization vectors in mixed $e^+e^- \to \tilde{\chi}^-_1\tilde{\chi}^+_2/\tilde{\chi}^-_2\tilde{\chi}^+_1$ pair-production [7].

Defining the $\tilde{\chi}^-_i$ production angle with respect to the electron flight-direction by Θ, the helicity amplitudes can be derived from (11). While electron and positron helicities are opposite to each other in all amplitudes, the $\tilde{\chi}^-_i$ and $\tilde{\chi}^+_j$ helicities are in general not correlated due to the non-zero masses of the particles; amplitudes with equal $\tilde{\chi}^-_i$ and $\tilde{\chi}^+_j$ helicities vanish only $\propto m_{\tilde{\chi}^\mp_{i,j}}/\sqrt{s}$ for asymptotic energies. Denoting the electron helicity by the first index, and the $\tilde{\chi}^-_i$ and $\tilde{\chi}^+_j$ helicities by the remaining two indices, λ_i and λ_j, respectively, the helicity amplitudes $T(\sigma;\lambda_i,\lambda_j) = 2\pi\alpha\langle\sigma;\lambda_i\lambda_j\rangle$ are given as follows [9],

$$\begin{aligned}\langle +;++\rangle &= -\left[Q_{RR}\sqrt{1-\eta^2_+} + Q_{RL}\sqrt{1-\eta^2_-}\right]\sin\Theta \\ \langle +;+-\rangle &= -\Big[Q_{RR}\sqrt{(1+\eta_+)(1+\eta_-)} \\ &\quad +Q_{RL}\sqrt{(1-\eta_+)(1-\eta_-)}\Big](1+\cos\Theta) \\ \langle +;-+\rangle &= +\Big[Q_{RR}\sqrt{(1-\eta_+)(1-\eta_-)} \\ &\quad +Q_{RL}\sqrt{(1+\eta_+)(1+\eta_-)}\Big](1-\cos\Theta) \\ \langle +;--\rangle &= +\left[Q_{RR}\sqrt{1-\eta^2_-} + Q_{RL}\sqrt{1-\eta^2_+}\right]\sin\Theta\end{aligned} \tag{16}$$

and

$$\begin{aligned}\langle -;++\rangle &= -\left[Q_{LR}\sqrt{1-\eta^2_+} + Q_{LL}\sqrt{1-\eta^2_-}\right]\sin\Theta \\ \langle -;+-\rangle &= +\Big[Q_{LR}\sqrt{(1+\eta_+)(1+\eta_-)} \\ &\quad +Q_{LL}\sqrt{(1-\eta_+)(1-\eta_-)}\Big](1-\cos\Theta) \\ \langle -;-+\rangle &= -\Big[Q_{LR}\sqrt{(1-\eta_+)(1-\eta_-)} \\ &\quad +Q_{LL}\sqrt{(1+\eta_+)(1+\eta_-)}\Big](1+\cos\Theta) \\ \langle -;--\rangle &= +\left[Q_{LR}\sqrt{1-\eta^2_-} + Q_{LL}\sqrt{1-\eta^2_+}\right]\sin\Theta\end{aligned} \tag{17}$$

where $\eta_\pm = \lambda^{1/2}(1,\mu^2_i,\mu^2_j) \pm (\mu^2_i - \mu^2_j)$ with the 2-body phase-space function $\lambda(1,\mu^2_i,\mu^2_j) = [1-(\mu_i+\mu_j)^2][1-(\mu_i-\mu_j)^2]$ and the reduced masses $\mu^2_i = m^2_{\tilde{\chi}^\pm_i}/s$. From these amplitudes the $\tilde{\chi}^-_i\tilde{\chi}^+_j$ production cross sections and the left–right asymmetries can be determined.

3.1 Production cross sections

The unpolarized differential cross section is given by the average/sum over the helicities:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Theta}(e^+e^- \to \tilde{\chi}^-_i\tilde{\chi}^+_j) = \frac{\pi\alpha^2}{32s}\lambda^{1/2} \times \sum_{\sigma\lambda_i\lambda_j} |\langle\sigma;\lambda_i\lambda_j\rangle|^2 \tag{18}$$

where λ is the two-body phase space function introduced above. Carrying out the sum, the following expression for the cross section in terms of the quartic charges can be derived:

$$\begin{aligned}&\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\Theta}(e^+e^- \to \tilde{\chi}^-_i\tilde{\chi}^+_j) \\ &= \frac{\pi\alpha^2}{2s}\lambda^{1/2}\Big\{\left[1-(\mu^2_i-\mu^2_j)^2 + \lambda\cos^2\Theta\right]Q_1 \\ &\quad +4\mu_i\mu_j Q_2 + 2\lambda^{1/2}Q_3\cos\Theta\Big\}\end{aligned} \tag{19}$$

[1] When expressed in terms of the fundamental SUSY parameters, they do depend nevertheless indirectly on $\cos\Phi_\mu$ through $\cos 2\phi_{L,R}$, in the same way as the masses depend indirectly on this parameter.

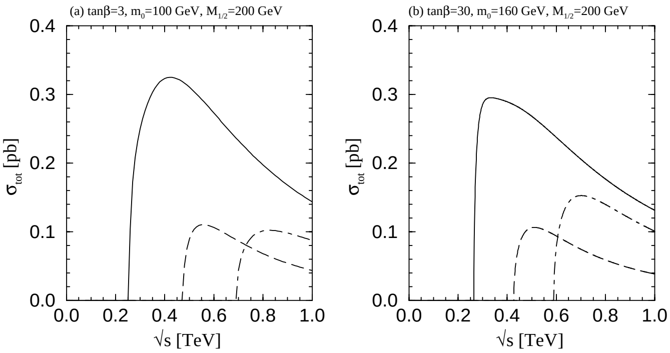

Fig. 2. The cross sections for the production of charginos as a function of the c.m. energy **a** with the set [$\tan\beta = 3, m_0 = 100\,\text{GeV}, M_{1/2} = 200\,\text{GeV}$] and **b** with the set [$\tan\beta = 30, m_0 = 160\,\text{GeV}, M_{1/2} = 200\,\text{GeV}$]: solid line for $\tilde{\chi}_1^-\tilde{\chi}_1^+$ production, dashed line for $\tilde{\chi}_1^-\tilde{\chi}_2^+$ production, and dot-dashed line for $\tilde{\chi}_2^-\tilde{\chi}_2^+$ production

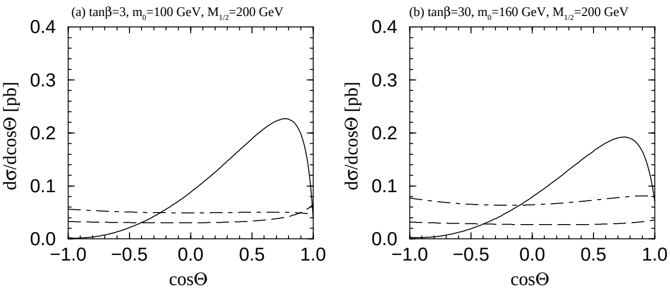

Fig. 3. The angular distributions as a function of the scattering angle at a c.m. energy of 800 GeV **a** with the set [$\tan\beta = 3, m_0 = 100\,\text{GeV}, M_{1/2} = 200\,\text{GeV}$] and **b** with the set [$\tan\beta = 30, m_0 = 160\,\text{GeV}, M_{1/2} = 200\,\text{GeV}$]: solid line for $\tilde{\chi}_1^-\tilde{\chi}_1^+$ production, dashed line for $\tilde{\chi}_1^-\tilde{\chi}_2^+$ production, and dot-dashed line for $\tilde{\chi}_2^-\tilde{\chi}_2^+$ production

If the production angle could be measured unambiguously on an event-by-event basis, the quartic charges could be extracted directly from the angular dependence of the cross section at a single energy. After integration over the production angle Θ, the total cross section still depends on Q_3 since t-channel sneutrino exchange gives rise to a non-linear forward–backward asymmetric angular dependence.

The total production cross section is shown in Fig. 2 as a function of the c.m. energy for a fixed sneutrino mass. The sneutrino mass is assumed to be predetermined from direct production $e^+e^- \to \tilde{\nu}_e\overline{\tilde{\nu}_e}$. The curves, which should be interpreted as characteristic examples, are based on the two CP-invariant mSUGRA scenarios introduced in [10]. They correspond to a small and a large $\tan\beta$ solution for the universal gaugino and scalar masses:

$$\begin{aligned} \boldsymbol{RR1}: \text{small} \quad \tan\beta &= 3 : (m_0, M_{\frac{1}{2}}) \\ &= (100\text{ GeV}, 200\text{ GeV}) \\ \boldsymbol{RR2}: \text{large} \quad \tan\beta &= 30 : (m_0, M_{\frac{1}{2}}) \\ &= (160\text{ GeV}, 200\text{ GeV}) \end{aligned} \tag{20}$$

Table 2. Gaugino and higgsino mass parameters, mass values of the charginos and the lightest neutralino, and of the sneutrino in the reference points of the mSUGRA scenarios introduced in [10]

$\tilde{m}$ [GeV]	$\boldsymbol{RR1}: \tan\beta = 3$	$\boldsymbol{RR2}: \tan\beta = 30$
M_2	152	150
μ	316	263
$\tilde{\chi}_1^\pm$	128	132
$\tilde{\chi}_2^\pm$	346	295
$\tilde{\chi}_1^0$	70	72
$\tilde{\nu}$	166	206

The induced chargino $\tilde{\chi}_{1,2}^\pm$, neutralino $\tilde{\chi}_1^0$, and sneutrino masses $\tilde{\nu}$ are collected in Table 2. The CP-phase Φ_μ is set to zero. The sharp rise of the production cross sections in Fig. 2 allows us to measure the chargino mass $m_{\tilde{\chi}_{1,2}^\pm}$ very precisely [4,11]. Figure 3 exhibits the angular distribution as a function of the scattering angle for the parameters of Table 2 at the c.m. energy 800 GeV. The peak in the near-forward region is due to the t-channel sneutrino exchange.

3.2 Left-right asymmetries

Switching the longitudinal electron polarization yields a left-right (LR) asymmetry $\mathcal{A}_{LR}$, defined as

$$\mathcal{A}_{LR} = \frac{1}{4}\sum_{\lambda_i\lambda_j}\left[|\langle +;\lambda_i\lambda_j\rangle|^2 - |\langle -;\lambda_i\lambda_j\rangle|^2\right]/\mathcal{N} \tag{21}$$

with the normalization

$$\mathcal{N} = \frac{1}{4}\sum_{\lambda_i\lambda_j}\left[|\langle +;\lambda_i\lambda_j\rangle|^2 + |\langle -;\lambda_i\lambda_j\rangle|^2\right] \tag{22}$$

The LR asymmetry $\mathcal{A}_{LR}$ can be readily expressed in terms of the quartic charges,

$$\begin{aligned} \mathcal{A}_{LR} = 4\Big\{&[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda\cos^2\Theta]Q_1' \\ &+4\mu_i\mu_j Q_2' + 2\lambda^{1/2}\cos\Theta Q_3'\Big\}/\mathcal{N} \end{aligned} \tag{23}$$

with, correspondingly,

$$\begin{aligned} \mathcal{N} = 4\Big\{&[1 - (\mu_i^2 - \mu_j^2)^2 + \lambda\cos^2\Theta]Q_1 \\ &+4\mu_i\mu_j Q_2 + 2\lambda^{1/2}\cos\Theta Q_3\Big\} \end{aligned} \tag{24}$$

In Fig. 4 the LR asymmetries are depicted as a function of the scattering angle for the parameters of Table 2 at the c.m. energy 800 GeV. The large negative asymmetry for $\tilde{\chi}_1^-\tilde{\chi}_1^+$ production in the forward direction is due to the t-channel sneutrino exchange which affects only the cross section for left-handed electron beams.

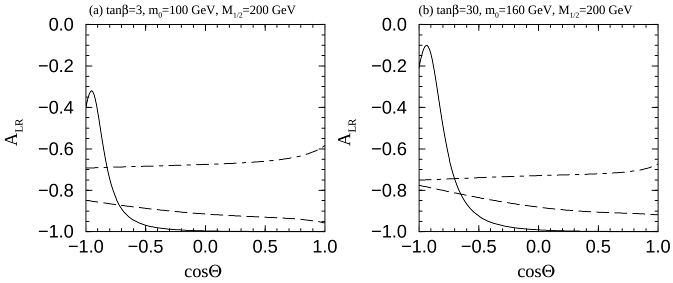

Fig. 4. The LR asymmetries as a function of the scattering angle at a c.m. energy of 800 GeV **a** with the set $[\tan\beta = 3, m_0 = 100\,\mathrm{GeV}, M_{1/2} = 200\,\mathrm{GeV}]$ and **b** with the set $[\tan\beta = 30, m_0 = 160\,\mathrm{GeV}, M_{1/2} = 200\,\mathrm{GeV}]$: solid line for $\tilde{\chi}_1^-\tilde{\chi}_1^+$ production, dashed line for $\tilde{\chi}_1^-\tilde{\chi}_2^+$ production, and dot-dashed line for $\tilde{\chi}_2^-\tilde{\chi}_2^+$ production

3.3 Polarization vectors

The polarization vector $\mathcal{P} = (\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N)$ is defined in the rest frame[2] of the chargino $\tilde{\chi}_i^-$. $\mathcal{P}_L$ denotes the component parallel to the $\tilde{\chi}_i^-$ flight direction in the c.m. frame, $\mathcal{P}_T$ the transverse component in the production plane, and $\mathcal{P}_N$ the component normal to the production plane. These three components can be expressed by helicity amplitudes in the following way:

$$\begin{aligned}
\mathcal{P}_L &= \frac{1}{4}\sum_{\sigma=\pm}\left\{|\langle\sigma;++\rangle|^2 + |\langle\sigma;+-\rangle|^2 - |\langle\sigma;-+\rangle|^2 - |\right. \\
&\quad \left.\times\,\langle\sigma;--\rangle|^2\right\}/\mathcal{N} \\
\mathcal{P}_T &= \frac{1}{2}\mathrm{Re}\left\{\sum_{\sigma=\pm}[|\langle\sigma;++\rangle\langle\sigma;-+\rangle^* + |\langle\sigma;--\rangle\right. \\
&\quad \left.\times\,\langle\sigma;+-\rangle^*]\right\}/\mathcal{N} \\
\mathcal{P}_N &= \frac{1}{2}\mathrm{Im}\left\{\sum_{\sigma=\pm}[|\langle\sigma;--\rangle\langle\sigma;+-\rangle^* - |\langle\sigma;++\rangle\right. \\
&\quad \left.\times\,\langle\sigma;-+\rangle^*]\right\}/\mathcal{N}
\end{aligned} \tag{25}$$

The longitudinal, transverse and normal components of the $\tilde{\chi}_i^-$ polarization vector can easily be obtained from the helicity amplitudes. Expressed in terms of the quartic charges, they read:

$$\begin{aligned}
\mathcal{P}_L &= 4\left\{2(1-\mu_i^2-\mu_j^2)\cos\Theta Q_1' + 4\mu_i\mu_j\cos\Theta Q_2'\right. \\
&\quad \left.+\lambda^{1/2}[1+\cos^2\Theta-(\mu_i^2-\mu_j^2)\sin^2\Theta]Q_3'\right\}/\mathcal{N} \\
\mathcal{P}_T &= -8\left\{[(1-\mu_i^2+\mu_j^2)Q_1' + \lambda^{1/2}Q_3'\cos\Theta]\mu_i\right. \\
&\quad \left.+(1+\mu_i^2-\mu_j^2)\mu_j Q_2'\right\}\sin\Theta/\mathcal{N} \\
\mathcal{P}_N &= 8\lambda^{1/2}\mu_j\sin\Theta Q_4/\mathcal{N}
\end{aligned} \tag{26}$$

[2] Axis $\hat{z}\|L$ in the flight direction of $\tilde{\chi}_i^-$, $\hat{x}\|T$ rotated counterclockwise in the production plane, and $\hat{y} = \hat{z}\times\hat{x}\|N$.

Table 3. Values of the CP-odd quartic charge Q_4 and the normal polarization component $\mathcal{P}_N$ for $\sqrt{s} = 800$ GeV and three production angles Θ. The reference points ***RR*1/2** have been defined earlier; the CP angle Φ_μ is chosen $\pi/2$

	Θ	Q_4	$\mathcal{P}_N$
***RR*1**	$\pi/4$	-0.199	-0.333
	$\pi/2$	-0.073	-0.246
	$3\pi/4$	-0.044	-0.129
***RR*2**	$\pi/4$	-0.026	-0.043
	$\pi/2$	-0.010	-0.027
	$3\pi/4$	-0.006	-0.013

The longitudinal and transverse components are P-odd and CP-even, and the normal component is P-even and CP-odd.

The normal polarization component can only be generated by complex production amplitudes, see [12]. Non-zero phases are present in the fundamental SUSY parameters if CP is broken in the supersymmetric interaction. Also the non-zero width of the Z boson and loop corrections generate non-trivial phases; however, the width effect is negligible for high energies and the effects due to radiative corrections are small as well. So, the normal component is effectively generated by the complex SUSY couplings. The bilinear charges are real in the diagonal modes (1,1) and (2,2) so that the normal polarization vanishes. But, the non-diagonal modes (1,2) and (2,1) may have non-vanishing normal polarization components, determined by the quartic charge

$$\begin{aligned}
Q_4 &= \frac{1}{32c_W^4 s_W^4}\left[D_Z^2(2s_W^4 - s_W^2 + \frac{1}{4})\right. \\
&\quad \left.+D_Z D_{\tilde{\nu}} c_W^2(s_W^2 - \frac{1}{2})\right] \\
&\quad \times\sin 2\phi_L\sin 2\phi_R\sin(\beta_L-\beta_R+\gamma_1-\gamma_2)
\end{aligned} \tag{27}$$

When combined with the relation in (15), the unknown sign of the product $\sin 2\phi_L$ with $\sin 2\phi_R$ can be eliminated. The ensuing coefficient $\tan(\beta_L-\beta_R+\gamma_1-\gamma_2)$ depends on $\sin\Phi_\mu$ as evident from the definition of the four phase angles $\{\gamma_1,\gamma_2,\beta_L,\beta_R\}$ in (7). The normal polarization component is generally small. Since CP violating effects like $\mathcal{P}_N$ or Q_4 are proportional to the imaginary part of $M_2\mu m_W^2\sin 2\beta$, i.e.,the product of the $\mathcal{M}_C$ matrix elements, they vanish for asymptotically large values of $\tan\beta$. A few numerical examples are displayed for $\sqrt{s} = 800$ GeV in Table. 3, based on the two reference points ***RR*1** and ***RR*2** introduced earlier, and the CP phase $\Phi_\mu = \pi/2$.

Due to the two escaping LSPs, it is difficult to measure the normal polarization components. Nevertheless, CP-odd observables, that are indirectly related to Q_4 and $\mathcal{P}_N$, may be constructed to measure the sign of $\sin\Phi_\mu$ – the only parameter left to be determined. An example is the triple product of the initial electron momentum and the two final-state lepton momenta in the $\tilde{\chi}^{\pm}_{1,2}$ leptonic decays, $\mathcal{O}_3 = \mathrm{sgn}\,[\mathbf{p}_{e^-} \cdot (\mathbf{p}_{l^-} \times \mathbf{p}_{l^+})]$. This observable depends on the phenomenological analysis powers κ_1 and $\bar{\kappa}_2$ which, however, can be measured experimentally; therefore, the analysis does not require any knowledge of the structure of the neutralino sector. In particular, the observable $\mathcal{O}_3$, based on single-particle momenta of the two parent charginos, does not depend on potentially CP-violating couplings in the decay processes, see [13].

4 Observables and extraction of SUSY parameters

4.1 Phenomenological analysis

The pair-production of the charginos χ^-_i and $\tilde{\chi}^+_j$ is characterized by the chargino masses $m_{\tilde{\chi}^{\pm}_{1,2}}$ and the two mixing angles, ϕ_L and ϕ_R [besides the sneutrino mass $m_{\tilde{\nu}}$]. These quantities can be determined from three chargino pair-production cross sections and three LR asymmetries. Nevertheless, we assume the sneutrino mass to be measured independently in sneutrino pair-production.

The chargino masses $m_{\tilde{\chi}^{\pm}_{1,2}}$ can be determined very precisely at the per-mille level near the threshold where the production cross sections $\sigma(e^+e^- \to \tilde{\chi}^-_1\tilde{\chi}^+_1)$, $\sigma(e^+e^- \to \tilde{\chi}^-_1\tilde{\chi}^+_2)$ and $\sigma(e^+e^- \to \tilde{\chi}^-_2\tilde{\chi}^+_2)$ rise sharply with the chargino velocities.

Combining the energy variation of the cross sections with the measurements of the LR asymmetries, the two mixing angles ϕ_L, ϕ_R and $\cos\Phi_\mu$ can be extracted. Based on the first parameter set in (20), we will demonstrate that the three chargino production modes enable us to extract *unambiguously* the values of two cosines, $\cos 2\phi_L$ and $\cos 2\phi_R$, by measuring only their production cross sections and LR asymmetries with longitudinally-polarized electron beams. In the mSUGRA scenario implemented with radiative corrections, the parameter set (20) with $\tan\beta = 3$ leads to the following values for cross sections and asymmetries at the c.m. energy $\sqrt{s} = 800$ GeV:

$$\begin{aligned}\boldsymbol{RR1}: \sigma_{tot}(1,1) &= 0.197\text{pb}, \quad \sigma_{tot}(1,2) = 0.068\text{pb}, \\ \sigma_{tot}(2,2) &= 0.101\text{pb} \quad A_{LR}(1,1) = -0.995, \\ A_{LR}(1,2) &= -0.911, \quad A_{LR}(2,2) = -0.668 \end{aligned} \tag{28}$$

From now on we will interpret this set as experimentally "measured values", neglecting experimental errors for the time being. The set[3] will be exploited to pin down a unique point in the $\{\cos 2\phi_L, \cos 2\phi_R\}$ plane which leads back, in combination with the masses, to a unique solution for the fundamental SUSY parameters.

Figure 5 exhibits the contours in the $\{\cos 2\phi_L, \cos 2\phi_R\}$ plane for the "measured values" of the cross sections, $\sigma_{tot}(1,1)$, $\sigma_{tot}(1,2)$ and $\sigma_{tot}(2,2)$ and the LR asymmetries, $A_{LR}(1,1)$, $A_{LR}(1,2)$ and $A_{LR}(2,2)$ in the diagonal and mixed pair-production processes. In this special case, the $\tilde{\chi}^-_1\tilde{\chi}^+_1$ mode alone gives one solution and the other contours cross at the same point which is marked by a fat dot. In general, the cross section and asymmetry contours intersect twice for each (ij) pair combination. However, combining the observables of the lightest pair (11) with the second lightest pair (12) already leads to a unique solution [discarding accidental cases of zero measure] that can be cross-checked again by measuring the (2,2) observables:

$$[\cos 2\phi_L, \cos 2\phi_R] = [0.67, 0.85] \tag{29}$$

If the three measurements could not be interpreted by a single $[\cos 2\phi_L, \cos 2\phi_R]$ solution, the basic set-up of the 2×2 SUSY chargino system would have to be extended.

In practice, the errors in the observables $m_{\tilde{\chi}^{\pm}_{1,2}}$ and $\cos 2\phi_{L,R}$ must be analyzed experimentally and the migration to the fundamental SUSY parameters must be studied properly. This is, however, beyond the scope of the purely theoretical analysis in this paper.

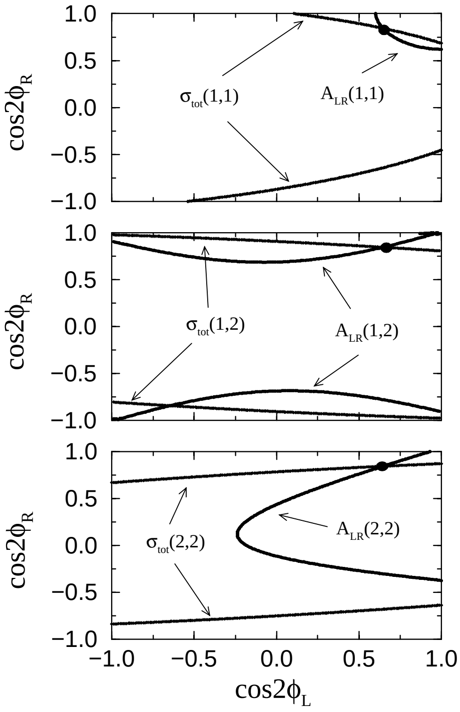

Fig. 5. Contours in the $\{\cos 2\phi_L, \cos 2\phi_R\}$ plane for "measured values" of the total cross section $\sigma_{tot}(i,j)$ and the LR asymmetry $A_{LR}(i,j)$ for $\tilde{\chi}^-_i\tilde{\chi}^+_j$ $[i,j = 1,2]$ production. The upper frame describes the (1,1) mode, the central frame the (1,2) mode and the lower frame the (2,2) mode. The fat dot in each figure marks the common crossing point of the contours

4.2 Fundamental SUSY parameters

From the two masses $m_{\tilde{\chi}^{\pm}_1}$ and $m_{\tilde{\chi}^{\pm}_2}$ and the mixing angles $\cos 2\phi_L$ and $\cos 2\phi_R$, the basic SUSY parameters $\{\tan\beta, M_2, |\mu|, \cos\Phi_\mu\}$ can be derived unambiguously in the following way.

[3] The large $\tan\beta$ set in (20) leads to the same conclusions.

(i) $\tan\beta$: The value of $\tan\beta$ is uniquely determined in terms of two chargino masses and two mixing angles

$$\tan\beta = \sqrt{\frac{4m_W^2 + (m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm})(\cos 2\phi_R - \cos 2\phi_L)}{4m_W^2 - (m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm})(\cos 2\phi_R - \cos 2\phi_L)}} \quad (30)$$

For $\cos 2\phi_R$ larger (smaller) than $\cos 2\phi_L$ the value of $\tan\beta$ is larger (smaller) than unity.

(ii) $M_2, |\mu|$: Based on the definition $M_2 > 0$, the gaugino mass parameter M_2 and the modulus of the higgsino mass parameter read as follows:

$$\begin{aligned} M_2 &= \tfrac{1}{2}\sqrt{2(m^2_{\tilde{\chi}_2^\pm} + m^2_{\tilde{\chi}_1^\pm} - 2m_W^2) - (m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm})(\cos 2\phi_R + \cos 2\phi_L)} \\ |\mu| &= \tfrac{1}{2}\sqrt{2(m^2_{\tilde{\chi}_2^\pm} + m^2_{\tilde{\chi}_1^\pm} - 2m_W^2) + (m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm})(\cos 2\phi_R + \cos 2\phi_L)} \end{aligned} \quad (31)$$

(iii) $\cos\Phi_\mu$: The sign of μ in CP-invariant theories and, more generally, the cosine of the phase of μ in CP-non-invariant theories is determined as well by the $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$ masses and $\cos 2\phi_{L,R}$: Using (30) and (31), $\cos\Phi_\mu$ is obtained from

$$\cos\Phi_\mu = \frac{(m^2_{\tilde{\chi}_2^\pm} - m^2_{\tilde{\chi}_1^\pm})^2 - (M_2^2 - |\mu|^2)^2 - 4m_W^2(M_2^2 + |\mu|^2) - 4m_W^4\cos^2 2\beta}{8m_W^2 M_2|\mu|\sin 2\beta} \quad (32)$$

As a result, the fundamental SUSY parameters $\{\tan\beta, M_2, \mu\}$ in CP-invariant theories, and $\{\tan\beta, M_2, |\mu|\cos\Phi_\mu\}$ in CP-noninvariant theories, can be extracted unambiguously from the observables $m_{\tilde{\chi}^\pm_{1,2}}$, $\cos 2\phi_R$, and $\cos 2\phi_L$. The final ambiguity in $\Phi_\mu \leftrightarrow 2\pi - \Phi_\mu$ in CP-noninvariant theories must be resolved by measuring observables related to the normal $\tilde{\chi}_1^-$ or/and $\tilde{\chi}_2^+$ polarization in non-diagonal (1,2) chargino pair-production [7].

5 Comment on the neutralino sector

Due to the large ensemble of four neutralino states $[\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0]$ in the bino-wino-higgsino sector, the analysis is much more complex in this case. Nevertheless, after measuring the SU(2) gaugino mass M_2 and the higgsino mass parameter μ (including the phase) in the chargino sector, the symmetric MSSM neutralino mass matrix

$$\mathcal{M}_N = \begin{pmatrix} |M_1|\mathrm{e}^{i\Phi_1} & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ & M_2 & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ & & 0 & -|\mu|\mathrm{e}^{i\Phi_\mu} \\ & & & 0 \end{pmatrix} \quad (33)$$

involves only two unknown parameters: the modulus and the phase of the (complex) U(1) gaugino mass $M_1 = |M_1|e^{i\Phi_1}$.

Deferring the detailed analysis for a CP-noninvariant theory to a sequel of this paper, [14], the analysis of CP invariant theories is much less complex[4]. Since $\mathcal{M}_N^2$ is symmetric and positive, an orthogonal matrix $\mathcal{N}$ can be constructed that transforms $\mathcal{M}_N^2$ to a positive diagonal matrix. This mathematical problem can be solved analytically.

Introducing the four-set of invariants associated with $\mathcal{M}_N^2$,

$$\begin{aligned} A &= \mathrm{tr}\mathcal{M}_N^2 = M_1^2 + M_2^2 + 2\mu^2 + 2m_Z^2 \\ B &= \tfrac{1}{2}[(\mathrm{tr}\mathcal{M}_N^2)^2 - \mathrm{tr}\mathcal{M}_N^4] \\ &= (\mu^2 + m_Z^2)^2 + 2\mu^2(M_1^2 + M_2^2) + M_1^2 M_2^2 \\ &\quad + 2m_Z^2[c_W^2 M_1^2 + s_W^2 M_2^2 - \mu\sin 2\beta(c_W^2 M_2 + s_W^2 M_1)] \\ C &= \tfrac{1}{6}[(\mathrm{tr}\mathcal{M}_N^2)^3 - 3\mathrm{tr}\mathcal{M}_N^2\mathrm{tr}\mathcal{M}_N^4 + 2\mathrm{tr}\mathcal{M}_N^6] \\ &= \mu^2[\mu^2(M_1^2 + M_2^2) + m_Z^4\sin^2 2\beta + 2M_1^2 M_2^2] \\ &\quad + m_Z^4[c_W^4 M_1^2 + 2c_W^2 s_W^2 M_1 M_2 + s_W^4 M_2^2] \\ &\quad + 2m_Z^2\mu^2(c_W^2 M_1^2 + s_W^2 M_2^2) \\ &\quad - 2m_Z^2\mu\sin 2\beta[c_W^2 M_2(\mu^2 + M_1^2) + s_W^2 M_1(\mu^2 + M_2^2)] \\ D &= \det\mathcal{M}_N^2 \\ &= \mu^4 M_1^2 M_2^2 + m_Z^4\mu^2[c_W^4 M_1^2 + 2c_W^2 s_W^2 M_1 M_2 \\ &\quad + s_W^4 M_2^2]\sin^2 2\beta \\ &\quad - 2m_Z^2\mu^3 M_1 M_2[c_W^2 M_1 + s_W^2 M_2]\sin 2\beta \end{aligned} \quad (34)$$

the consistency condition

$$m^8_{\tilde{\chi}_1^0} - Am^6_{\tilde{\chi}_1^0} + Bm^4_{\tilde{\chi}_1^0} - Cm^2_{\tilde{\chi}_1^0} + D = 0 \quad (35)$$

must be fulfilled by the lowest of the eigenvalues $m^2_{\tilde{\chi}_1^0}$. Since $m_{\tilde{\chi}_1^0}$ can be measured precisely in chargino decays [4],

$$\tilde{\chi}_1^\pm \to W^\pm + \tilde{\chi}_1^0 \quad (36)$$

i.e.,with an error of $\mathcal{O}$ (100 MeV), (35) is a well-determined quadratic form that can be solved for M_1 up to a 2-fold ambiguity. Moreover, it has been shown in [15] that, in fact without using further experimental input, the ambiguity can be removed by linearizing the consistency conditions in $\mathcal{M}_N$ instead of $\mathcal{M}_N^2$ in a somewhat involved mathematical procedure.

6 Conclusions

We have analyzed how the parameters of the chargino system, the chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the size of the wino and higgsino components in the chargino wave-functions parameterized by two angles ϕ_L and ϕ_R, can be extracted from pair-production of the chargino states in e^+e^- annihilation. In addition to the three production cross sections, longitudinal electron polarization, which should be

[4] If M_1 and M_2 are real at the same time, which may be realized approximately in grand-unified scenarios, the subsequent analysis is modified only slightly to the extent that μ^{2N+1} is replaced by $|\mu|^{2N+1}\cos\Phi_\mu$ while even powers of μ are not altered except for the substitution $\mu^{2N} \to |\mu|^{2N}$.

realized at e^+e^- linear colliders, gives rise to three independent LR asymmetries. This method is independent of the chargino decay properties, i.e., the analysis is not affected by the structure of the neutralino sector which is very complex in extended supersymmetric theories while the chargino sector remains isomorphic to the simple form of the MSSM.

From the chargino masses $m_{\tilde{\chi}^\pm_{1,2}}$ and the two mixing angles ϕ_L and ϕ_R, the fundamental SUSY parameters $\{\tan\beta, M_2, \mu\}$ can be extracted in CP-invariant theories; in CP-noninvariant theories the modulus of μ and the cosine of the phase can be determined, leaving us with just a discrete two-fold ambiguity. The ambiguity can be resolved however by measuring the sign of observables related to the normal $\tilde{\chi}^\pm_{1,2}$ polarizations.

Moreover, from the energy distribution of the final particles in the decay of the charginos $\tilde{\chi}^\pm_1$, the mass of the lightest neutralino $\tilde{\chi}^0_1$ can be measured. This allows us to derive the parameter M_1 in CP-invariant theories so that the neutralino mass matrix, too, can be reconstructed in a model-independent way.

In summary: the measurement of the processes $e^+e^- \to \tilde{\chi}^-_i \tilde{\chi}^+_j$ provides a complete analysis of the fundamental SUSY parameters $\{\tan\beta, M_2, \mu\}$ in the chargino sector.

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